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On 12 June 1997 Vladimir Igorevich Arnol’d, one of the leading mathematicians of the present day, an academician of the Russian Academy of Sciences and President of the Moscow Mathematical Society, reached the age of 60. His papers on the theory of dynamical systems, classical and celestial mechanics, the theory of singularities, topology, real and complex algebraic geometry, symplectic, contact and projective geometry, hydrodynamics, the calculus of variations, differential geometry, potential theory, mathematical physics, the theory of superpositions, the history of mathematics, combinatorics, and in several other areas of mathematics and mechanics have won him world-wide recognition and they establish the current state of knowledge in many of these fields.

Vladimir Igorevich was born in Odessa, the son of Igor Vladimirovich Arnol’d, a well-known mathematician and specialist in number theory. From 1954 he studied at the Mechanics and Mathematics Faculty of Moscow State University [MGU] where he worked under the supervision of Kolmogorov, gaining a Ph.D. in 1961 and a Doctorate in 1963. From 1961 to 1986 he worked in the Mechanics and Mathematics Faculty of MGU and became a Professor in 1965. At present he is a chief scientific worker at the Steklov Institute of Mathematics of the Russian Academy of Sciences and at the same time a Professor at the Paris-Dauphine University.

His student papers [27], [31] that completed the solution (begun by Kolmogorov) of Hilbert's thirteenth problem brought him a world reputation: is it true that a real algebraic function $z(a, b, c)$, defined by the equation $z^7 + az^3 + bz^2 + cz + 1 = 0$, is not representable by superpositions of continuous functions of two variables? Nobody doubted that the answer to this question was positive, until Kolmogorov discovered that for $n > 3$ any continuous function of $n$ variables was representable by a superposition of functions of three variables. Vladimir Igorevich proved that the bound of three could be decreased, and functions of three variables reduced to superpositions of functions of two, thereby giving a negative answer to Hilbert's question. This result was the subject of Arnol’d’s Ph.D. dissertation.

In 1968 Arnol’d returned to the theory of superpositions in a new—an algebraic—formulation that came closest to the original stimulus for studying the problem.
He showed (see [87]) that the topological obstacle to the representation of a universal $n$-valued complex algebraic function $z(a_1, \ldots, a_{n-1})$, defined by the condition

$$z^n + a_1 z^{n-2} + a_2 z^{n-3} + \cdots + a_{n-1} = 0,$$

as a complete superposition of algebraic functions with a smaller number of variables lies in the cohomology group of the braid group on $n$ threads (or, equivalently, of the space of complex polynomials of degree $n$ without repeated roots). For $n = 2^k$ this representation is non-trivial, so that the function (*) does not decompose into a superposition of functions of $\leq n - 2$ variables. Later the cohomologies of braid groups were completely calculated by D. B. Fuchs, G. Segal and F. Cohen and found application, in particular, in papers by S. Smale and V. A. Vasil’ev in the theory of the complexity of computation and in interpolation theory. Prompted by these papers of Arnol’d, A. G. Khovanskii found a topological variant of Gauss’s theory of differentials.

These papers by Arnol’d laid the foundation for the topological theory of discriminants, that is, of subsets of function spaces that consist of functions with non-typical singularities. A fundamental role in this theory is played by the following observations of his (see [86], [233]). Instead of studying homology groups of a space of non-specific objects (such as polynomials without repeated roots), it is more convenient to consider the homologies (dual to those of Alexander) of the discriminant space. In fact, a set of non-specific objects is an open manifold without obvious geometric structure, whereas a discriminant is a particular stratified set, whose strata correspond to functions with abnormally complex or multiple singularities, and several homology classes can be successfully expressed in terms of these strata. The development of this theory has led, in particular, to the invention of invariants of finite order for knots and plane curves (see [283], [284], [292], [293].

Arnol’d’s paper on the cohomologies of coloured braids [80] initiated the topological theory of collections of planes (arrangements) that was carried forward by E. Brieskorn, P. Orlik and L. Solomon, M. Goresky and R. MacPherson, and several others. Unexpected applications of this work have since been found: for example, the fundamental correlation it has exposed between differential forms generating the ring of these homologies is one of the fundamental constituent steps in the construction of the Kontsevich integral for the invariants of knots.

Arnol’d is one of the creators of KAM (Kolmogorov–Arnol’d–Moser) theory. KAM theory is the theory of perturbations of quasiperiodic motions in dynamical systems. A quasiperiodic motion is the name given to a uniform motion along an irrational winding of a torus. For example, the phase space of a completely integrable Hamiltonian system with $n$ degrees of freedom is stratified into invariant $n$-dimensional tori. The motion on each torus takes place with constant speed, and in the general case on almost all tori this motion is quasiperiodic (the frequency ratios are incommensurable). Poincaré called the study of the behaviour of solutions of Hamiltonian systems close to integrability the fundamental problem of dynamics. The main difficulty with this problem are the so-called small denominators—integral linear combinations of frequencies that are close to zero, which appear in the denominators of terms of standard series of perturbation theory and make these series divergent.
The gap in the solution of ‘the fundamental problem of dynamics’ was closed by Kolmogorov in 1954. In his famous article “On conservation of conditionally periodic motions for a small change in Hamilton’s function”, Dokl. Akad. Nauk SSSR 98 (1954), 527–530, Kolmogorov showed that if, in a completely integrable Hamiltonian system, the frequencies of the motions on the torus depend in a non-degenerate way on the torus itself, then under any sufficiently small perturbation of the system a majority of the tori with a quasiperiodic motion do not collapse and are only slightly deformed, because the motions on the perturbed tori remain quasiperiodic. To prove this theorem Kolmogorov used a powerful new method of high-speed sequential changes of variables (going back to Newcomb’s procedure in classical perturbation theory and to Newton’s method of tangents for the estimation of the roots of equations, generalized by Kantorovich for application to function spaces). The so-called ‘quadratic convergence’ of Kolmogorov’s method makes it possible to overcome the influence of small denominators.

Kolmogorov did not publish a detailed proof of his theorem. The first detailed exposition of Kolmogorov’s method was given by Arnol’d in his 1959 thesis (published in 1961, see [35]), in which this method was adapted to establish the analyticity of Denjoy’s homeomorphism, which associates with a rotation a diffeomorphism of the circle that is analytic close to the rotation. A complete proof of Kolmogorov’s 1954 theorem itself was first published by Arnol’d in 1963 in [45].

Kolmogorov’s 1954 paper laid the foundation of the theory of quasiperiodic motions in non-integrable systems. The next stage in the development of this theory is linked with Arnol’d’s achievements, the most important of which are the generalization of Kolmogorov’s theorem to Hamiltonian systems with degeneracies and the discovery of a universal mechanism of instability of Hamiltonian systems with several degrees of freedom.

In many integrable Hamiltonian systems one meets the so-called limiting degeneracy—the dimensionality of individual tori is found to be less than the number of degrees of freedom. The simplest example is the position of equilibrium of a system with one degree of freedom surrounded by invariant circles. Another type of degeneracy is the so-called characteristic degeneracy, occurring when some of the frequencies of the perturbed quasiperiodic motion tend to zero together with the perturbation. For example, when examining a system that depends on a parameter that changes in a slowly periodic way, a small frequency of change of this parameter will constitute a perturbation.

In a series of papers [36], [37], [41], [43], [47] during 1961–1963, Arnol’d generalized Kolmogorov’s theorem to a range of classes of systems with degeneracies and, in particular, obtained definitive solutions to some classical problems of Hamiltonian dynamics: Birkhoff’s problem on the stability of a fixed point of an area-preserving mapping of the plane onto itself in the general elliptic case, the problem of the stability of a position of equilibrium of a Hamiltonian system with two degrees of freedom in the general elliptic case, the problem of permanent adiabatic invariance of a variable action in a system with \(n < 2\) degrees of freedom, and that of the stability of planetary systems. As to the last problem, in which both types of singularity are present, Arnol’d showed that in planetary systems with sufficiently small planets quasiperiodic motions make up a set of positive Lebesgue measure. These results were the subject matter of Arnol’d’s D.Sc. dissertation.
In a Hamiltonian system that is close to integrable, the variable actions remain permanently close to their initial values for the trajectories that lie on invariant tori (and in the general case the latter fill the greater part of phase space). For trajectories lying in the resonance zones between the tori, this is not necessarily so (provided the number of degrees of freedom is greater than two). In [50] Arnol'd constructed a famous example of a system that satisfies all the conditions of the proposition concerning the survival of a majority of the non-perturbed tori and in which for some initial conditions the variable actions in fact depart greatly from their initial values. This phenomenon was subsequently called by physicists Arnol'd's diffusion and was studied both numerically and analytically in papers by many authors. The rapidity of the diffusion in Arnol'd's example is exponentially small in comparison with the perturbation. In 1971 Nekhoroshev proved that the speed of Arnol'd's diffusion is always exponentially small if the non-perturbed Hamiltonian does not belong to a certain set of infinite codimension. The mechanism of the diffusion described in Arnol'd's 1964 paper has a universal quality and acts hypothetically in systems of a general nature.

Among Arnol'd's other achievements associated with the problem of small denominators are the study of evolution in multifrequency systems as they pass through resonance [54], the foundation of a theory of neighbourhoods of holomorphic subvarieties [120], and an investigation of invariant tori in so-called reversible systems [191], [211].

At the present time KAM theory has become an extensive area of the theory of dynamical systems, drawing in ideas, methods and results connected with small perturbations and quasiperiodic motions in various dynamical systems, and it has numerous applications in mechanics, celestial mechanics, plasma physics, nuclear physics, and other branches of science.

Arnol'd proposed a new method in hydrodynamics, having shown that Euler's equation for an ideal fluid is an equation of geodesics on a group of diffeomorphisms that preserve the volume element (with respect to the right-invariant metric defined by the energy of the fluid) [64]. One of the applications of this approach by Arnol'd is the impossibility of a reliable long-range weather forecast: the fact that the curvatures of the group of diffeomorphisms are negative implies an exponential instability of fluid flows or atmospheric motions [3].

Vladimir Igorevich classified the stationary motions of a fluid in a plane and in space and described sufficient conditions for their stability [62], [65]. In particular, he showed that plane flows without points of inflection of the velocity profile are stable, and that there was a generalization of Rayleigh's theorem on linear stability. His method, known as A-stability, has been extensively developed in hydrodynamics.

Arnol'd has introduced the concept of the asymptotic Hopf invariant that has allowed the evaluation of geometric characteristics of magnetic fields by means of their topological invariants [111]. This has opened up new horizons for the application of topological methods in the classical hydrodynamics of an ideal fluid and in magnetohydrodynamics, and has provided the basis for numerous generalizations.

In the theory of the magnetic dynamo, Arnol'd is responsible for the definition of ABC-flows (Arnol'd–Beltrami–Childress), which provide one of the fundamental models of the dynamo [58], [178], for the construction of the fast kinematic dynamo on a 3-dimensional Riemannian manifold (with Zel'dovich, Ruzmaikin and
Sokolov [160]), and also of dynamos obtained from a geodesic flow on surfaces of constant negative curvature [25].

At the end of the 1960s Vladimir Igorevich became interested (as he recalled during long conversations with R. Thom and B. Morin) in a broad new class of problems connected with the actions of infinite-dimensional continuous groups, such as groups of diffeomorphisms on various function spaces. These problems subsequently led to the modern theory of singularities and its applications. One of the first of his papers in this field is the article [94], which introduced the concept of versal deformation, which now occupies a key place in many departments of analysis and differential equations. If, for example, there is a family of matrices that depends smoothly on some parameters, then, although it is possible to convert each individual matrix to Jordan normal form, it is in general impossible to do this by means of transformations that depend smoothly on the parameters. The simplest normal forms of such families are connected with transversals to the orbits of the associated action. This finite-dimensional case, which is important in its own right in the theory of the stability of differential equations, became a model example of the study of the stratification of singularities and the geometry of bifurcation diagrams for a large number of applied problems that incorporate parameters.

It is hard to find a branch of the modern theory of dynamical systems to which Arnol'd has not made important contributions. In “Lectures on bifurcations and versal families”, delivered at Katsiveli in 1971, he introduced the fundamental concepts and the terminology that the modern theory of bifurcations uses (see [102]). His research student A. N. Shoshtitaishvili proved the principle of reduction, according to which when investigating bifurcations of singular points of vector fields the system may be restricted to its central manifold. With the aid of this principle, all bifurcations of equilibrium positions in typical single-parameter families were reduced to ones already known. The investigation of bifurcations in two-parameter families was begun by R. I. Bogdanov in Arnol’d’s seminar and has continued up to the present time. While investigating the loss of stability of auto-oscillations in 1977, Arnol’d discovered that during the passage of a pair of multipliers across a strong resonance there arise non-local bifurcations associated with the appearance and destruction of separatrix polygons [132]. This work was continued by E. Khorozov, Kh. Zholondek, and others.

In 1970 Arnol’d proposed a research programme of local problems in analysis [83]. This programme gave some structure towards a general approach, according to which, when investigating a new phenomenon, it is necessary to examine ab initio aspects of the general case, then degeneracies of codimension 1, and so on. At the same time he proved the algebraic undecidability of the stability problem and the problem of the topological classification of singular points of vector fields [88].

In 1969, when refereeing A. D. Bryuno’s doctoral thesis, Arnol’d proposed a geometric explanation of the divergence of normalizing series for vector fields close to singular points, when the spectrum of the linear part is pathologically near to a countable number of resonance planes. The explanation lies in ‘materialization of resonances’: the appearance of a countable number of invariant manifolds in the complex domain. The presence of such varieties in an arbitrary neighbourhood of a singular point is an obstacle to linearizability. To explain the nature of this obstacle
in three dimensions, Arnol’d created a theory of normal forms of neighbourhoods of elliptic curves [120].

Under his influence A. A. Davidov found normal forms of singular points of implicit differential equations. In 1985 Arnol’d revealed a link between the theory of such equations and fast–slow systems [198], [217]. At the same time he found normal forms of singular points on slow surfaces of dimension two.

Arnol’d is one of the founders of the modern theory of singularities of smooth mappings. While investigating the asymptotic theory of quasiclassical solutions of problems in quantum optics, he constructed a classification (to within the action of the natural groups of smooth changes of the variables) of degenerate singular points of differentiable functions and Lagrange manifolds, and elucidated the link between these objects and the geometry of regular polyhedra and the crystallographic symmetry groups [101]. Thus, simple singularities (those not dependent on continuous parameters) of functions in \( \mathbb{R}^n \) can be naturally classified according to the types \( A_k, D_k, E_6, E_7, E_8 \), while the analogous singularities of functions on a manifold with a boundary (arising naturally in a problem about bypassing an obstacle) can be classified according to the types \( B_k, C_k, F_4 \), [145], [147].

Problems on the reduction of geometric objects (functions, vector fields, differential forms, and so on) to a normal form with the aid of a diffeomorphism had been investigated by many mathematicians, including Poincaré, but even here Arnol’d invented the strongest method—spectral sequences of quasihomogeneous vector fields ([113], [119]), which is effective where the classical methods turn out to be powerless. To the problem of the classification of singularities he applied Newton’s method of polyhedra ([108], [115]). Subsequently D. N. Bernshtein, A. N. Varchenko, A. G. Kushnirenko and A. G. Khovanskii used Newton polyhedra for the efficient calculation of the topological characteristics of singularities and algebraic varieties, and this led to an extensive theory of Newton polyhedra.

Connected with this same problem on the classification of singularities, Vladimir Igorevich began an investigation into the asymptotic behaviour of oscillating integrals in the method of stationary phase with degenerate singular points. He showed [105] that answers can be expressed in terms of the Coxeter numbers of the corresponding groups generated by reflections, having thus linked the behaviour of the integrals with the monodromy operator of the singularity. This link was later explained by B. Malgrange and gave rise to the investigations of J. Steenbrink, A. N. Varchenko and others into mixed Hodge structures in vanishing homologies.

The ‘strange duality’ noted in [115] between numerical characteristics of unimodal singularities was the first example of the ‘mirror symmetry’ that became extraordinarily popular later.

In papers by Arnol’d and his students singularities and typical perestroikas were classified: of caustics and wave fronts, Maxwell sets, vector fields, evolutes, maximum functions, boundaries of natural domains in function spaces (of stability, ellipticity, hyperbolicity, inability to oscillate, and so on), and of many other objects (see [6], [15], [23], [124].

Mathematicians have always been interested in how plane algebraic curves can be discovered. The ancient Greeks were very familiar with the conic sections. Newton analysed what kinds of cubic curves can occur and later those of degrees 4 and 5 were described. Curves of even higher degree, however, had proved resistant
to all study. Hilbert had suspected that not all qualitative pictures of algebraic curves that are a priori possible can actually be realized, and that there must exist some mysterious prohibitions. He therefore included the question of the topological classification of algebraic curves of stipulated degree in the list of his famous problems (at number 16). In spite of individual efforts (D. Hilbert, A. Harnack, K. Rohn, I. G. Petrovskii, Petrovskii and O. A. Oleinik, D. A. Gudkov), up to the time of Arnol’d’s paper the situation had remained enigmatic. This mysteriousness had reached its highest point in 1970 when Gudkov enumerated all admissible topological types of mutual arrangement of ovals of projective curves of degree 6, and showed that all $M$-curves (that is, curves with the maximum possible number of connected components (ovals) that is not prohibited by Harnack’s inequality) of even degree $2k$, $k \leq 3$, satisfy the following relation: the number $P$ of their even ovals (that is, ovals lying inside an even number of other ovals) minus the number $L$ of their odd ones is congruent to $k^2$ modulo 8.

Arnol’d proved Gudkov’s congruence for arbitrary $k$ (though only modulo 4), having linked this problem to modern 4-dimensional topology and the arithmetic of integral bilinear forms [95]. In brief, this approach consists in the following.

Let $A \subset \mathbb{RP}^2$ be a non-singular $M$-curve of even degree $2k$, $CA$ its complexification, and $p : Y \to \mathbb{CP}^2$ a two-sheeted covering of the space $\mathbb{CP}^2$, ramified along $CA$. $Y$ is a smooth compact complex manifold. The curve $A$ divides $\mathbb{RP}^2$ into two parts—orientable and non-orientable. We shall put $\Pi := p^{-1}(\text{orientable part})$. This is a two-dimensional compact orientable manifold. Let $[\Pi]$ and $[CA]$ be classes of the manifolds $\Pi$ and $CA$ in the group $H_2 := H_2(Y, \mathbb{Z})/\text{Tors}$. Let $\tau : Y \to Y$ be an involution that transposes the sheets of the covering $p$. We shall define on the group $H_2$ a bilinear form $\Phi$ by the rule $\Phi(\alpha, \beta) = (\tau^* \alpha, \beta)$, where $(\cdot, \cdot)$ is the index of the intersection.

The desired congruence is now proved by the following chain of lemmas.

(a) $\tau_*[CA] = [CA], \tau_*[\Pi] = -[\Pi]$ (by definition);
(b) $([CA], [CA]) = 2k^2$ (Bézout’s theorem);
(c) $([\Pi], [\Pi]) = -\chi(\Pi) = -(P - L)$;
(d) the class $w \equiv [CA]$ is canonical for the form $\Phi$, that is, $\Phi(x, x) \equiv \Phi(w, x)$ (mod 2) for any $x \in H_2$. It follows from this that for any class $y \in H_2$ the class $w' \equiv w + 2y$ is also canonical and $\Phi(w', w') \equiv \Phi(w, w)$ (mod 2);
(e) (the key observation): the classes $[CA]$ and $[\Pi]$ are the same modulo 2, that is, there exists $y \in H_2$ such that $[\Pi] = [CA] + 2y$. In particular, $\Phi([CA], [CA]) \equiv \Phi([\Pi], [\Pi])$ (mod 8).

On the other hand, $\Phi([CA], [CA]) = 2k^2$ (from (a) and (b)), and $\Phi([\Pi], [\Pi]) = 2(P - L)$ (from (a) and (c)), so that finally we obtain the congruence $2k^2 \equiv 2(P - L) \pmod 8$.

This paper effectively initiated a new science—real algebraic geometry—which was developed in papers by V. A. Rokhlin, V. M. Kharlamov, O. Ya. Viro, V. V. Nikulin, E. I. Shustin, and many others. Gudkov’s congruence at its full extent (modulo 8) was proved after some time by Rokhlin using Arnol’d’s approach.

In one of his last papers Poincaré discovered that an area-preserving diffeomorphism of a ring that rotates the boundary circles in different directions must
have at least two fixed points. Poincaré did not succeed in proving his last geometric theorem. The proof of the actual fact that had been found by Poincaré was soon obtained by Birkhoff, but further development of the subject came to a complete halt. Only Arnol’d managed to take the next step. He guessed that symplectomorphisms of symplectic manifolds that are homologous to the identity mapping must have many fixed points (not less than the sum of the Betti numbers of the manifold). Arnol’d’s conjecture, which he formulated in its simplest case at the International Mathematics Congress at Moscow in 1966, was proved in papers by C. Conley and E. Zehnder, Yu. V. Chekanov, A. Floer, M. Chaperon, J.-C. Sikorav, ... and led to important progress in the calculus of variations, to the creation of symplectic topology, and to the discovery of Floer’s homologies and quantum cohomologies.

In [66] Arnol’d gave a classical interpretation of the one-dimensional class of cohomologies of a Lagrange submanifold of the symplectic space $\mathbb{R}^{2n}$, shortly before this was defined by V. P. Maslov as a class, dual to a set (with coordinates defined in a natural way) of singular points of a Lagrange projection of general position. Arnol’d proved that this class is induced under a Gaussian transformation from the Grassmann homologies of all Lagrange planes in $\mathbb{R}^{2n}$, having thus extended the construction to the case of manifolds not of general position and proved the invariance of the class being examined under continuous deformations of a Lagrange manifold. In fact, this paper by Arnol’d introduced this class—the Maslov index—into modern topology and the theory of Lie groups and initiated research into the topology of Lagrange Grassmannians and the theory of characteristic classes dual to singular sets of smooth manifolds.

Subsequently this index arose in the theory he founded of Lagrange and Legendre cobordisms [154], [155].

In a classical field, such as the theory of Newtonian potential, Arnol’d discovered a non-trivial generalization of the theorem of Newton and Ivory on the attraction of spheres and ellipsoids to the case of arbitrary hyperbolic surfaces [168], and also in magnetic fields induced by electric currents in hyperboloids of one sheet [177], [246].

While reading Newton’s *Principia*, he perceived in the proof of a theorem on the non-integrability of plane ovals ideas from the theory of monodromy, and later these allowed him to obtain many-dimensional generalizations of this theorem [220], [246].

In a short survey it is not possible to mention all Vladimir Igorevich’s other results. A still incomplete list would include ‘Arnol’d’s inequalities’ for topological characteristics of real algebraic varieties that generalize the Petrovskii–Oleinik inequalities [141], a *global* variant of Liouville’s theorem on completely integrable systems [49], papers on the theory of Berry phases and the quantum Hall effect [99], [295], a description of singularities in a collision-free powdered medium [165], the classification of normal forms of Poisson structures in $\mathbb{R}^2$ and $\mathbb{R}^3$ [200], [218], and much more.

Arnol’d is a remarkable teacher, who has created a school of specialists in the theory of singularities, dynamical systems and symplectic geometry that is known all over the world. Among his direct students are about 40 Ph.D.s and 5 D.Sc.s, although the number of professional mathematicians who can count him as their teacher, who have taken their first steps in science in societies and seminars he has directed or who have been indebted to him for key ideas in their papers,
is very much greater. He possesses the rare gift of finding among the simplest of materials new and elegant problems that are capable of capturing the interest of young researchers: let us recall, for instance, his recent papers on invariant plane curves or on the calculation of snakes [262], [265]. His specialist and compulsory courses, reports at conferences and lectures for beginners are invariably sparkling and swiftly bring the listener close to the heart of new theories that have previously seemed impossibly abstruse. In practice all papers by Vladimir Igorevich are a starting point for further research by other writers. The same may be said about the continuously flowing stream of new problems emanating from him: see, for example [118], [133], [156], [226], [250], [271].

The whole world has been taught by Arnol’d’s books. His textbooks “Mathematical methods of classical mechanics”, “Ordinary differential equations” and “Additional chapters in the theory of ordinary differential equations” have become an essential part of mathematical education.

The driving force behind Arnol’d’s research has been an inexhaustible interest in everything, a wish to find out ‘What’s inside there?’, to experience over and over again the joy of discovery: see [253], [269], [270], [276], [277], [286], [288], [305], [315]. His incredible energy, erudition and mathematical style, his ability to go straight to the heart of a task and find new perspectives, links to other areas of mathematics, and interesting new problems enliven any company of mathematicians and few of them remain unimpressed.

Vladimir Igorevich is the author of more than 300 publications in various fields of mathematics, including more than 20 books.

For his research, he has been awarded the Moscow Mathematical Society prize (1958), the Lenin prize (1965, jointly with A. N. Kolmogorov), the Crafoord prize of the Swedish Academy (1982), the Lobachevskii prize of the Russian Academy of Sciences (1992) and the Harvey prize (1994).

Arnol’d has been elected a Member of the USSR/Russian Academy of Sciences (1990, having been a Corresponding Member since 1984), a Member of the Russian Academy of Natural Sciences (1991), a foreign Member of the USA National Academy (1983), of the French Academy of Sciences (1984), of the USA Academy of Sciences and Arts (1987), of the Royal Society of London (1988), of the Accademia dei Lincei of Rome (1988), and of the European Academy (1991); he is also an Honorary Member of the London Mathematical Society. He has honorary Doctorates from the Pierre and Marie Curie University of Paris (1979), Warwick University (1988), Utrecht University (1991), Bologna University (1991), the Complutense University of Madrid (1994), and Toronto University (1997).

He was an invited lecturer at four International Mathematics Congresses (1958, 1966, 1974 and 1983) and at the first European Mathematics Congress (1992), see [32], [69], [114], [192], [284]; at three of these he gave a plenary lecture.

Many constructs, theorems, phenomena, hypotheses and theorems in mathematics are named after him because in spite of his favourite jest he has the most direct hand in their creation. A far from complete list would include KAM-theory, Arnol’d diffusion, ABC streams, Arnol’d’s languages (in perturbation theory), A-stability (in hydrodynamics), Arnol’d’s hypothesis (in symplectic topology), the Maslov–Arnol’d characteristic classes, Arnol’d’s spectral sequence,
the Hilbert–Arnol’d problem, Arnol’d’s inequalities and congruences in real algebraic geometry, and a countless number of Arnol’d theorems, lemmas and methods in various theories.

At the present time Vladimir Igorevich is Vice-President of the International Mathematics Union, one of the founders and the virtual ideological leader of the Independent Moscow University, editor-in-chief of the journal Funktsional. Anal. i Prilozhen. [Functional Analysis and its Applications] of the Russian Academy of Sciences, a member of the Editorial Board of numerous Russian and foreign scientific publications.

In addition he is supervisor to 7 or 8 research students in Russia and France; during the spring semester he has a second teaching post in Paris and in the autumn gives lecture courses at the Independent University. He is frequently away at international scientific meetings and his appearance at any mathematics society or conference is for them a reliable remedy against stiffness and passivity. We wish Vladimir Igorevich many splendid new discoveries, good students, health and happiness.

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